PHYSICS OF Q^2-DEPENDENCE IN THE NUCLEON'S $G_1(x, Q^2)$ STRUCTURE FUNCTION SUM RULE

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ABSTRACT

I discuss in this talk the physics of the Q^2 dependence of the $G_1(x, Q^2)$ structure function sum rule. For $Q^2 > 3$ GeV^2, the Q^2 variation is controlled by pure QCD radiative corrections. For $0.5 < Q^2 < 3$ GeV^2, the twist-four contribution becomes significant, but stays perturbative. For Q^2 below ~ 0.05, the sum rule is determined by low-energy theorems. The rapid change of the sum rule between 0.05 and 0.5 GeV^2 signals the transition between parton and hadron degrees of freedom.

In polarized electron or muon scattering on a polarized nucleon target, one measures the nucleon tensor,

$$W_{\mu\nu} = \frac{1}{2} \sum_n (2\pi)^3 \delta^4(P + q - P_n)$$
$$\times \langle PS|J_\mu(0)|n\rangle\langle n|J_\nu(0)|PS\rangle \ , \tag{1}$$

where $|PS\rangle$ is the ground state of the nucleon with momentum P^μ and polarization S^μ, $|n\rangle$ are the excited states of the nucleon after absorbing the virtual photon of momentum q^μ, and J_μ is the usual electromagnetic current of the nucleon, which are composed of quark fields. The spin-dependent part of the tensor is known to depend on two Lorentz scalar structure functions $G_1(Q^2, \nu)$ and $G_2(Q^2, \nu)$,

$$W_{\mu\nu}|_{\text{spin-depen.}} = -i\epsilon_{\mu\nu\alpha\beta}q^\alpha \left(\frac{G_1}{M^2}S^\beta + \frac{G_2}{M^4} \left(S^\beta \nu M - P^\beta(S \cdot q) \right) \right) \tag{2}$$

where $Q^2 = -q^2$ and $\nu M = P \cdot q$.

Although we shall not always work in the Bjorken limit, it turns out convenient to replace variable ν by x: $x = Q^2/2M\nu$. The drawback of doing this is that the whole photo-production region shrinks to a point $x = 0$ and $Q^2 = 0$. However, for our purpose it is not a problem. I assume from now on that $G_1(Q^2, x)$ is measured to a good precision in low and intermediate Q^2 regions. This may turn out to be the biggest assumption of my talk. I certainly hope this can be done in the future at CEBAF and other places.

Two interesting sum rules exist for G_1 at large and small Q^2, respectively. The deep-inelastic sum rule is defined at $Q^2 \to \infty$ limit,

$$\Gamma = \lim_{Q^2 \to \infty} \int_0^1 g_1(x, Q^2)dx \ ,$$
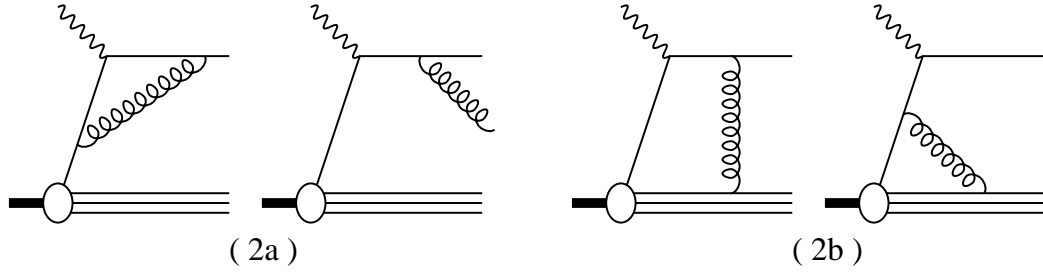$$= \frac{1}{2} \sum_i e_i^2 \Delta q_i \ , \tag{3}$$

FIG. 1. a). QCD radiative corrections to deep-inelastic scattering. b). Higher-twist corrections to deep-inelastic scattering.

where $g_1(x, Q^2) = (\nu/M)G_1(\nu, Q^2)$ is the scaling function and Δq_i is the axial charge for quark flavor i, which is defined by,

$$\langle PS|\bar{\psi}_i\gamma_\mu\gamma_5\psi_i|PS\rangle = 2\Delta q_i S_\mu \ . \tag{4}$$

The Bjorken sum rule relates $\Gamma^p - \Gamma^n$ to the neutron β-decay constant g_A [1] and Ellis-Jaffe sum rules refer to a model prediction for Γ^p and Γ^n made by Ellis and Jaffe [2].

The Drell-Hearn-Gerasimov (DHG) sum rule is a sum rule for $G_1(\nu, Q^2)$ at the real photon point $Q^2 = 0$ [3]. For simplicity of discussion, I view the sum rule as the limit of $Q^2 \to 0$,

$$\lim_{Q^2 \to 0} \int_{\nu_{\text{in}}}^{\infty} \frac{d\nu}{\nu} G_1(\nu, Q^2) = -\frac{1}{4}\kappa^2 \ , \tag{5}$$

where ν_{in} is the inelastic threshold and κ is the anomalous magnetic moment of the nucleon. Using the scaling function, I can write,

$$\int_0^1 dx g_1(x, Q^2)^{\text{inelastic}} = -\frac{\kappa^2}{8}\frac{Q^2}{M^2} + \mathcal{O}\left(\left(\frac{Q^2}{M^2}\right)^2\right) \ , \tag{6}$$

for small Q^2. The question I want to address below is what physics controls the variation of the sum rule between the large and small Q^2 limits.

First let me consider deep-inelastic sum rules at large but finite Q^2. There are two types of QCD corrections to the $Q^2 \to \infty$ limit. The first is the QCD radiative corrections shown in Fig. 1(a), which take into account the effects of hard gluons in the hard process. The second is the higher-twist corrections shown in Fig. 1(b), which are basically initial and final state interactions between the active quark and the remnants of the target. For example, the Bjorken sum rule with these corrections reads,

$$\int_0^1 g_1^{p-n}(x, Q^2)dx = \frac{g_A}{6}\left(1 - \frac{\alpha_s(Q^2)}{\pi} - \cdots\right) + \frac{\mu_4^{p-n}(Q^2)}{Q^2} + \cdots \tag{7}$$

where the terms in the bracket represent radiative effects and μ_4^{p-n} is the nucleon matrix elements of some twist-two, three, and four operators [4].

A number of comments can be made about the sum rule in Eq. (7):

- Theoretically there is an ambiguity in separating out contributions of different twists. This was first recognized by A. Mueller [5]. The problem is that the perturbative series for radiative corrections is not convergent. It is a non-Borel-summable series. Thus the result obtained by Ellis and Karliner by comparing the data with the four-loop prediction should be taken with a grain of salt, particularly at low Q^2 [6]. I have recently outlined a solution to the problem [7], but I cannot talk about it here due to time limitation.

- The sum rule *must* include the elastic contribution as it becomes important below $Q^2 = 2$ GeV^2. The reason is obvious: the sum rule is derived from operator product expansion and one gets an operator product only when all intermediate states are summed over. If one is still not convinced, consider the nucleon is a point-like particle, then the only contribution to the sum rule is elastic scattering [8]. For the nucleon, the elastic contribution to Γ is,

$$\Gamma^{\text{elastic}} = \frac{1}{2}F_1(F_1 + F_2) - \frac{1}{8M^2}F_2^2 Q^2 \ . \tag{8}$$

where F_1 and F_2 are the usual Dirac and Pauli form factors.

- Higher-twist contributions have been estimated in the MIT bag model [4] and in QCD sum rule approach [9],

$$
\begin{aligned}
\mu_4^{p-n} &= 0.031 M^2 \,, && \text{(Bag)} \\
\mu_4^{p-n} &= -0.023 M^2 \,. && \text{(QSR)}
\end{aligned}
\tag{9}
$$

Here the infrared renormalon problem has been ignored. The two estimates differ in sign. This shows that we are not yet confident in calculating higher-twist matrix elements. However, it is quite clear that the size of the higher-twist contribution is small. It contributes at 10% level at $Q^2 = 2$ GeV^2, and becomes negligible at $Q^2 = 10$ GeV^2.

The above discussion shows that $\Gamma(Q^2)$ *changes very little from* $Q^2 = \infty$ *down to* $Q^2 = 0.5$ GeV^2. Radiative effects are on the order of 10% to 20% in the entire region. The twist-four effects are important only in the range $Q^2 \sim 0.5 - 3$ GeV^2. Again, their contribution is not overwhelming.

Now let me turn to the sum rule at $Q^2 \sim 0$. The DHG sum rule certainly needs to be tested. Its validity tells us whether there is a subtraction constant in the dispersion relation, whether the sum rule is convergent, and whether there are fixed pole contributions, etc. The detailed mechanism for sum rule saturation is also interesting. In particular, there are indications that the Δ excitation exhausts major part of the sum rule.

One can generalizes the DHG sum rule to small Q^2 by writing a low energy expansion,

$$
\int_0^1 dx g_1(x, Q^2)^{\text{inelastic}} = -\frac{\kappa^2}{8}\frac{Q^2}{M^2} + \alpha \left(\frac{Q^2}{M^2}\right)^2 + \cdots
\tag{10}
$$

where α is a parameter which can be calculated for instance in chiral perturbation theory. It is also interesting to test this type of generalized sum rule.

One interesting question is how to connect the DHG sum rule to the deep-inelastic sum rule. This question is first studied by Anselmino, Ioffe, and Leader [10], and the result has been quoted by many authors. Unfortunately, their study is wrong. They neglected the elastic contribution when interpolating high and low Q^2 sum rules and thus got the incorrect conclusion that Γ_p/Q^2 has to change sign at some intermediate Q^2. The sign change was considered as mysterious. As I said before, as Q^2 decreases, the high Q^2 side physics is controlled by twist expansion which, by definition, contains the elastic contribution.

Once the elastic contribution is included, we have at low Q^2,

$$
\begin{aligned}
\Gamma_p(Q^2) &= \Gamma_p(Q^2)^{\text{elastic}} + \Gamma_p(Q^2)^{\text{inelastic}} \\
&= 1.396 - 8.631 Q^2 + \alpha Q^4 + \cdots
\end{aligned}
\tag{11}
$$

According to the above, $\Gamma_p(Q^2 \to 0) \to 1.396$, which is much larger than $\Gamma_p(Q^2 = 10$ GeV$^2)$ $= 0.136$. Clearly, $\Gamma_p(Q^2)$ drops very quickly as Q^2 increases due to the large coefficient of the Q^2 term. In fact, if one neglects the higher order terms, $\Gamma_p(Q^2)$ drops to the level at $Q^2 = 10$ GeV^2 when $Q^2 = 0.15$ GeV^2. This behavior is certainly consistent with the small higher-twist effects at moderate Q^2. However, beyond that, the $Q^2 \sim 0$ behavior says nothing about the size of higher twist effects, contrary to many claims in the literature.

Certainly, the change between $Q^2 \sim 0.05$ and $Q^2 = 0.5$ is interesting. We do not have reliable theoretical prediction in the region. However, we believe that the transition between hadronic and partonic description of scattering occurs in this region. Since the transition is likely smooth, I think nothing drastic happens for $\Gamma_p(Q^2)$ other than a smooth connection between low and high Q^2 limits.

To summarize the above discussion, the physics of Q^2 variation of $\Gamma_p(Q^2)$ can be roughly divided into four regions. For $Q^2 > 3$ GeV^2, the Q^2 variation is controlled by QCD radiative corrections. In $0.5 < Q^2 < 3$ GeV^2, the twist-four contribution is important. Below ~ 0.05 GeV^2, the Q^2 variation is determined by low energy theorems. In $0.05 < Q^2 < 0.5$ GeV^2, parton-hadron transition happens. A rough sketch for the sum rule variation (the solid line) is shown in Fig. 2. The dotted line represents an extrapolation from high energy and the dash-dotted line an extrapolation from low energy.

Let me emphasize the importance of getting data in the region $0.5 < Q^2 < 3$ GeV^2. The sum rule here can be constructed with future resonance data from CEBAF and low x data from SLAC or HERMES. It allows one to extract the matrix element f,

$$
\langle PS | g \bar{\psi} \tilde{F}^{\mu\nu} \gamma_\nu \psi | PS \rangle = 2 f S^\mu \,.
\tag{12}
$$

f is very interesting from the nucleon's structure point of view. In fact, the sign of f determines roughly whether the color magnetic field $\mathbf{B}$ in the polarized nucleon is pointing to the direction of the spin or opposite.

Thus one can learn a lot of physics by measuring the first moment of the G_1 structure function at low and intermediate Q^2. I urge experimenters go ahead to take some good data on G_1.
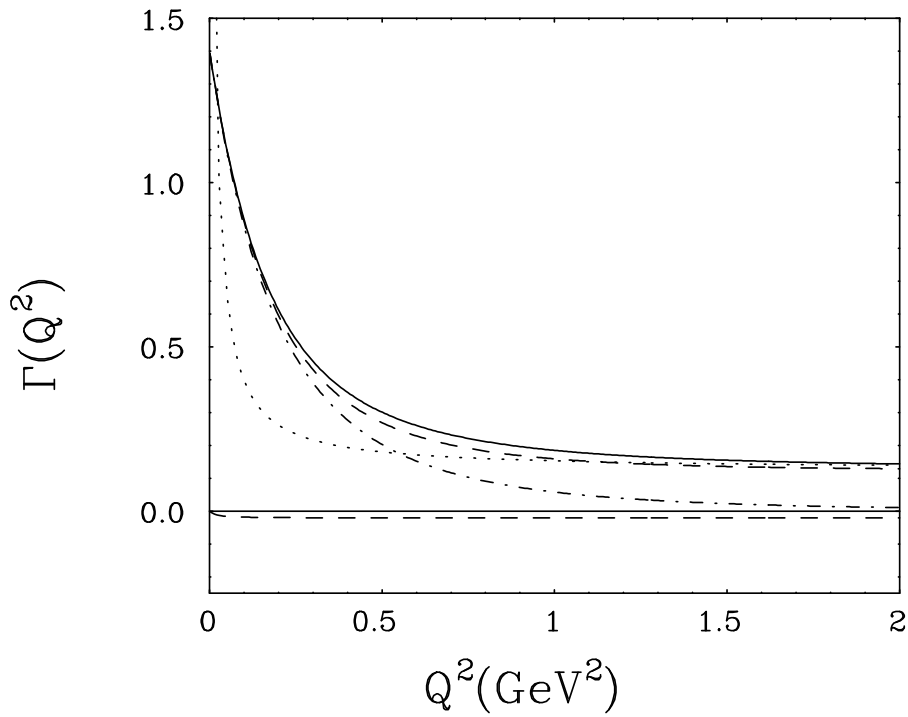
FIG. 2. Schematic Q^2-dependent of the first moment of G_1 structure function.

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